B.Sc. (Maths) part II

paper IV

Topic :- Reduction of a force system to a force and Couple

Force :- Def - Force is an external effort in the form of push or pull which (i) produces motion in body (ii) stops a moving body (iii) changes the direction of motion of a body.

Force = Mass x acceleration

$F = m \times f$

Couple : when two equal and unlike parallel forces acting on a body then this forces are called Couple.

By a action of Couple force we can twist the body round without moving any direction.

Theorem :- prove that any system of Coplaner forces acting on a rigid body is equivalent to a single force acting at an arbitrary point in the plane of forces together with couple.

Soln:— Let the system of force acting at a points $P_1, P_2, \ldots$ be respectively $\vec{F_1}, \vec{F_2} \ldots$

Let O be any arbitrary point introduce the two forces $\vec{F_1}$, and $-\vec{F_1}$ at O. These being in equilibrium among themselves, do not alter the effect of given force.

The forces $\vec{F}$ at P_1 and $-\vec{F_1}$ at O constitute a couple of moment $\vec{OP_1} \times \vec{F_1}$. Thus the force $\vec{F_1}$ at O together with a couple of moment $\vec{OP_1} \times \vec{F_1}$

Similar result hold good for other forces of the system acting at different points of rigid body.

Hence the whole system of forces is equivalent to forces $\vec{F_1}, \vec{F_2} \ldots$ at O together with an equal number of couples of moments

$\vec{OP_1} \times \vec{F_1}$, $\vec{OP_2} \times \vec{F_2}$, ...

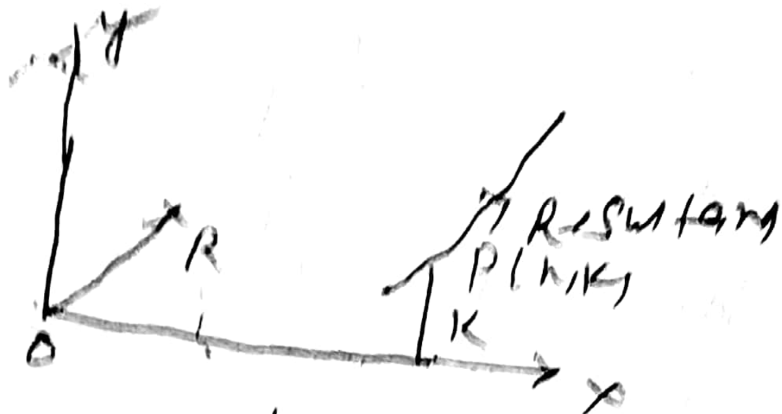
Now all the forces acting at O can be combined into a single resultant force $\vec{F} = \vec{F_1} + \vec{F_2} + \ldots$ and all the couple combined into single resultant couple

$$\vec{G} = \vec{OP_1} \times \vec{F_1} + \vec{OP_2} \times \vec{F_2} + \ldots$$

that is, the moment sum of the system about O.

<u>Theorem</u> :- Obtain the equation to the line of action of the resultant of a system of coplanar force.

We know that the system of coplanar forces acting on a rigid body can be reduced to a single force R acting at an arbitrary chosen point O, in the plane of the forces together with couple G.



Let P be any point (h,K) which lies on the resultant of the given system.

The moment of the system about P.

= the moment of the resultant about P = 0

i.e. $G + X.PQ - Y.OQ = 0$

i.e. $G + X.K - Y.h = 0$

Hence the locus of (h,K)

$$G + Xy - Yx = 0$$

which is the required eqn. of the line of action of the resultant force.